

SUPPLEMENTAL APPENDIX

Model equations and details for analysis of effects of temperature on emergence and seasonality of West Nile virus in California are shown below with symbols and values summarized in Table 1.

The mosquito vector population consists of uninfected (P_1) and infected (Q_1) eggs, and susceptible (S_1), incubating (infected, but not yet infectious) (E_1) and infectious (I_1) adult individuals. Adult mosquito population size is $N_1 = S_1 + E_1 + I_1$. The bird populations consist of susceptible (S_i), incubating (E_i), infectious (I_i), and immune (R_i) ($i = 2, 3, 4$) individuals. The total host population size is $N_i = S_i + E_i + I_i + R_i$ ($i = 2, 3, 4$). All the four populations are logistic in nature with carrying capacity K_i ($i = 1, 2, 3, 4$). The resulting model system of ordinary differential equations is

For the mosquito vectors,

$$\begin{aligned}\frac{dP_1}{dt} &= \frac{b_1 K_1}{N_1} (N_1 - q_1 I_1 - P_1) \\ \frac{dQ_1}{dt} &= \frac{b_1 K_1}{N_1} (q_1 I_1 - Q_1) \\ \frac{dS_1}{dt} &= \frac{b_1 K_1}{N_1} P_1 - \beta_{B1V} S_1 \frac{I_2}{N_2} - \beta_{B2V} S_1 \frac{I_3}{N_3} - \frac{d_1 N_1}{K_1} S_1 \\ \frac{dE_1}{dt} &= \beta_{B1V} S_1 \frac{I_2}{N_2} + \beta_{B2V} S_1 \frac{I_3}{N_3} - \left[\frac{d_1 N_1}{K_1} + \varepsilon_1 \right] E_1 \\ \frac{dI_1}{dt} &= \frac{b_1 K_1}{N_1} Q_1 + \varepsilon_1 E_1 - \frac{d_1 N_1}{K_1} I_1 \\ \frac{dN_1}{dt} &= \frac{b_1 K_1}{N_1} (P_1 + Q_1) - \frac{d_1 N_1}{K_1} (S_1 + E_1 + I_1)\end{aligned}$$

For avian host species #1,

$$\begin{aligned}\frac{dS_2}{dt} &= b_2 N_2 - \beta_{VB1} S_2 \frac{I_1}{N_1} - \beta_{BB} S_2 \frac{I_2}{N_2} - \beta_{BB} S_2 \frac{I_3}{N_3} - d_2 S_2 \frac{N_2}{K_2} \\ \frac{dE_2}{dt} &= \beta_{VB1} S_2 \frac{I_1}{N_1} + \beta_{BB} S_2 \frac{I_2}{N_2} + \beta_{BB} S_2 \frac{I_3}{N_3} - d_2 E_2 \frac{N_2}{K_2} - \varepsilon_2 E_2 \\ \frac{dI_2}{dt} &= \varepsilon_2 E_2 - (\gamma_2 + \mu_2) I_2 - d_2 I_2 \frac{N_2}{K_2} \\ \frac{dR_2}{dt} &= \gamma_2 I_2 - d_2 R_2 \frac{N_2}{K_2} \\ \frac{dN_2}{dt} &= b_2 N_2 - d_2 \frac{N_2^2}{K_2} - \mu_2 I_2\end{aligned}$$

For avian host species #2,

$$\begin{aligned}\frac{dS_3}{dt} &= b_3 N_3 - \beta_{VB2} S_3 \frac{I_1}{N_1} - d_3 S_3 \frac{N_3}{K_3} \\ \frac{dE_3}{dt} &= \beta_{VB2} S_3 \frac{I_1}{N_1} - d_3 E_3 \frac{N_3}{K_3} - \varepsilon_3 E_3 \\ \frac{dI_3}{dt} &= \varepsilon_3 E_3 - (\gamma_3 + \mu_3) I_3 - d_3 I_3 \frac{N_3}{K_3} \\ \frac{dR_3}{dt} &= \gamma_3 I_3 - d_3 R_3 \frac{N_3}{K_3} \\ \frac{dN_3}{dt} &= b_3 N_3 - d_3 \frac{N_3^2}{K_3} - \mu_3 I_3\end{aligned}$$

and for avian host species #3,

$$\frac{dN_4}{dt} = b_4 N_4 - d_4 S_4 \frac{N_4^2}{K_4}$$

It is possible to compute an analytical expression for the basic reproduction number, R_0 , for this model by writing $R_0 = R_0^{(V)} + R_0^{(H)}$. The first term, $R_0^{(V)}$, represents the direct transmission, which is the vertical transfer of West Nile virus from infectious mosquito mothers to their offspring, whereas the second term, $R_0^{(H)}$, is the indirect (vectorborne) transmission, which is the transmission between vectors mediated by bird hosts. To compute each component of R_0 , we apply the method described by van den Driessche and Watmough.⁴⁷ Calculating the basic reproduction number for the vertical transmission route, we find $R_0^{(V)} = \rho(F_v V_v^{-1})$, where

$$F_v = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ \frac{b_1 K_1}{N_1} & 0 & 0 \end{bmatrix},$$

$$V_v = \begin{bmatrix} \frac{b_1 K_1}{N_1} & 0 & -\frac{b_1 K_1}{N_1} q_1 \\ 0 & \frac{d_1 K_1}{N_1} + \varepsilon_1 & 0 \\ 0 & -\varepsilon_1 & \frac{d_1 K_1}{N_1} \end{bmatrix},$$

and $\rho(X)$ denotes the spectral radius of matrix X (i.e., the maximum of the absolute value of the eigenvalues of the next generation matrix $X = F_v V_v^{-1}$). For the case of horizontal transmission, we have $R_0^{(H)} = \rho(F_h V_h^{-1})$, where

$$F_h = \begin{bmatrix} 0 & 0 & 0 & \beta_{B1V} & 0 & \beta_{B2V} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \beta_{VB1} & 0 & \beta_{BB} & 0 & \beta_{BB} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \beta_{VB2} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix},$$

$$V_h = \begin{bmatrix} \frac{d_1 N_1}{K_1} + \varepsilon_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -\varepsilon_1 & \frac{d_1 N_1}{K_1} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{d_2 N_2}{K_2} + \varepsilon_2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \gamma_2 + \mu_2 + \frac{d_2 N_2}{K_2} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{d_3 N_3}{K_3} + \varepsilon_3 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -\varepsilon_3 & \gamma_3 + \mu_3 + \frac{d_3 N_3}{K_3} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{d_4 N_4}{K_4} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{d_4 N_4}{K_4} \end{bmatrix}$$

SUPPLEMENTAL APPENDIX TABLE 1
Model symbols, parameter values, and information source*

Symbol	Meaning	Value	Reference
b_1	Birth rate of vector	d_1	
b_2	Birth rate of host B ₁	d_2	
b_3	Birth rate of host B ₂	d_3	
b_4	Birth rate of host B ₃	d_4	
K_1	Carrying capacity of vector	Detail given above	
K_2	Carrying capacity of host B ₁	1,000	Arbitrary
K_3	Carrying capacity of host B ₂	1,000	Arbitrary
K_4	Carrying capacity of host B ₃	1,000	Arbitrary
$1/d_1$	Life span of vector	10 days based	68,69
$1/d_2$	Life span of host B ₁	3.72 years	70
$1/d_3$	Life span of host B ₂	1.23 years	70
$1/d_4$	Life span of host B ₃	0.79 years	70
q_1	Probability of vertical transmission	0.003	40,41,71
β_{B1V}	Adequate contact rate: host B ₁ to vector	$= \frac{f_{B1} \times r_{B1V}}{GP}$	
β_{B2V}	Adequate contact rate: host B ₂ to vector	$= \frac{f_{B2} \times r_{B2V}}{GP}$	
β_{VB1}	Adequate contact rate: vector to host B ₁	$= \frac{f_{B1} \times r_{VB1}}{GP}$	
β_{VB2}	Adequate contact rate: vector to host B ₂	$= \frac{f_{B2} \times r_{VB2}}{GP}$	
f_{B1}	Probability of feeding on host B ₁	$= \frac{N_2}{N_2 + N_3 + N_4}$	
f_{B2}	Probability of feeding on host B ₂	$= \frac{N_3}{N_2 + N_3 + N_4}$	
r_{B1V}	Probability of successful WNV transmission from Host B ₁ to vector per bite	0.46	72
r_{B2V}	Probability of successful WNV transmission from host B ₂ to vector per bite	0.19	72
r_{VB1}	Probability of successful WNV transmission from vector to host B ₁ per bite	1.0	37
r_{VB2}	Probability of successful WNV transmission from vector to host B ₂ per bite	1.0	37
β_{BB}	Probability of transmission from host to host	0.001	Arbitrary
$1/\varepsilon_1$	Extrinsic incubation period of vector	Detail given above	6
$1/\varepsilon_2$	Intrinsic incubation period of host B ₁	1 day	36,37
$1/\varepsilon_3$	Intrinsic incubation period of host B ₂	1 day	36,37
$1/\gamma_2$	Infectious period of host B ₁	1/5 days	36,37
$1/\gamma_3$	Infectious period of host B ₂	1/5 days	36,37
μ_2	Disease-related mortality rate of host B ₁	0.2/day	37
μ_3	Disease-related mortality rate of host B ₂	0.1/day	37

*WNV = West Nile virus.

SUPPLEMENTAL APPENDIX TABLE 2
Sensitivity analysis based on specific parameter*

Symbol	Range	PRCC	P
T	$[-3.29, 33.27]$	14.4349	$< 0.0001^\dagger$
d_1, b_1	$[1/21, 1/7]$	-3.8642	0.0001^\ddagger
d_2, b_2	$[1/(6.72 \times 365), 1/(0.75 \times 365)]$	0.2749	0.7836
d_3, b_3	$[1/(4.23 \times 365), 1/(0.75 \times 365)]$	-0.0669	0.9467
d_4, b_4	$[1/(3.79 \times 365), 1/(0.75 \times 365)]$	1.5807	0.1150
K_1	$[311.24, 112,892.06]$	22.0610	$< 0.0001^\dagger$
K_2	$[100, 10,000]$	-1.4635	0.1444
K_3	$[100, 10,000]$	1.6517	0.0996
K_4	$[100, 10,000]$	0.2802	0.7795
q_1	$[0.001, 0.008]$	7.2995	$< 0.0001^\dagger$
r_{B1V}	$[0.32, 0.59]$	1.1290	0.2598
r_{B2V}	$[0.05, 0.32]$	-0.0601	0.9521
r_{VB1}	1.0	-0.8208	0.4124
r_{VB2}	$[0.8, 1.0]$	0.1473	0.8830
β_{BB}	$[10^{-5}, 10^{-1}]$	53.1863	$< 0.0001^\dagger$
ε_2	$[0.5, 1]$	-0.4738	0.6360
ε_3	$[0.5, 1]$	-0.7929	0.4285
γ_2	$[1/7, 1/4]$	-6.7246	$< 0.0001^\dagger$
γ_3	$[1/6, 1/2]$	-0.3589	0.7199
μ_2	$[0.17, 0.37]$	-12.2846	$< 0.0001^\dagger$
μ_3	$[0.04, 0.17]$	0.5323	0.5949

*Sensitivity analysis based on specific parameter ranges (symbols defined in Supplemental Appendix Table 1) appears in Supplemental Appendix Table 2. Partial rank correlation coefficients were computed across ranges of parameters described in Appendix Table 1 to assess the significance of each parameter with respect to the fundamental reproductive ratio (R_0). PRCC = partial rank correlation coefficient.

$^\dagger P < 0.0001$.

$^\ddagger P = 0.0001$.